Metal oxide semiconductor structure and transistor behaviour using a single and simple graph \((Q\psi)\) which takes into account all the physical and electrical parameters

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Physical properties and electrical characteristics of the metal oxide semiconductor (MOS) structure and the MOS transistor depend on numerous parameters: doping and permittivity of the semiconductor, thickness, permittivity and charge of the oxide, temperature, and, obviously, gate, drain, and substrate voltages. In this paper, we propose a single and simple graph to visualize and highlight the impact of all these physical and electrical parameters. All main classical relationships can be easily deduced from this graph, which is an additional efficient tool to understand MOS device behaviors. © 2008 American Institute of Physics. [DOI: 10.1063/1.2885119]

I. INTRODUCTION

Complicated analytical expressions are necessary to understand the behavior of metal oxide semiconductor (MOS) structures and MOS transistors (MOSTs) as a function of the many parameters that impact the characteristics. However, the purely mathematical approach does not easily allow the individual influence of each of these various parameters to be synthesized. To make it easier, we propose a graphical representation of these equations. This approach allows visualization of the influence of all the parameters not only on the current of the transistor but also on electrostatic potentials and electric fields. These parameters are as follows: gate voltage, drain voltage, substrate voltage, doping of the substrate, thickness of oxide, substrate and oxide permittivities, oxide charge, and temperature. The effective mobility of carriers as well as MOST length and width will act as constant factors for the current.

The basic idea of this graph has been published previously.\(^{1}\) Here, we propose an extended version, with much more details, numerical values, physical explanations, and original extensions. We will first investigate the MOS structure, whose properties will be used to establish the MOS capacitance behavior, followed by the full MOST characteristics, and finally simplified ones. All physical constants, explanations, demonstrations, and classical relationships we shall refer to can be found in Ref. 2.

II. MOS STRUCTURE AND CAPACITANCE

A. Ideal MOS structure

For the sake of simplicity, let us choose a \(P\)-type semiconductor (denoted SC in the following) with a doping \(N_d\) and a permittivity \(\varepsilon_{SC}\) and start with the traditional expression of the charge per unit area \(Q_{SC}\) of this SC as a function of its surface potential \(\psi_s\) (cf., for example, Ref. 2, p. 368, or Ref. 3). For a \(P\)-type SC, \(Q_{SC}^A\) can be reduced to

\[
Q_{SC}^A = \left[ 2 \cdot \varepsilon_{SC} \cdot q \cdot N_d \left( \frac{kT}{q} \left( e^{\psi_s/kT} - 1 \right) + \psi_s - \psi_b \right) \right]^{1/2}
\]

The back side potential of the SC is assumed to be zero (see inset in Fig. 2). The potential \(\psi_b\) represents the potential difference between the intrinsic level and the Fermi level in the neutral material,

\[
\psi_b = (kT/q)\ln(N_d/n_i),
\]

where \(n_i\) is the intrinsic concentration (~10\(^{16}\) m\(^{-3}\) for silicon at 300 K), \(T\) the temperature, \(k\) the Boltzmann constant, and \(q\) the elementary charge; \(\psi_b\) is positive in a \(P\)-type SC. The total charge of the MOS structure is null, and the SC charge is located near the SC-oxide interface because of the electrostatic attraction due to the opposite charge in the metal.

This relationship is often represented using log-linear coordinates, but in the present case, its meaning will be clearer with linear scales. In Fig. 1, we can see a \(\sqrt{Q_s}\) variation (depleted zone charge) between two exponential zones: the first one corresponds to the positive charge of the holes (accumulation, \(\psi_s < 0\)) and the second one to the negative charge of the electrons (inversion, \(\psi_s > 2\psi_b\)).

\[Q_{SC}^A(\psi_s) = \left[ 2 \cdot \varepsilon_{SC} \cdot q \cdot \left( e^{\psi_b/kT} - 1 \right) + \psi_s \right]^{1/2}
\]

\[Q_{SC}^A(C/m^2) = \left[ 2 \cdot \varepsilon_{SC} \cdot q \cdot \left( e^{\psi_b/kT} - 1 \right) + \psi_s \right]^{1/2}
\]

FIG. 1. (Color online) SC total charge per unit area vs surface potential for silicon at 300 K.

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FIG. 2. (Color online) Graphical solution of SC charge per unit area as a function of gate voltage $V_G$, for an ideal MOS structure. In the present case, the total charge of the SC is $-0.0095 \text{ C/m}^2$ and the surface potential is 1.03 V.

This figure represents the first part of the MOS structure study. Obviously, the potential $\psi_s$ cannot be adjusted directly by the operator, which can only modify the gate potential $V_G$. For the purpose of the analysis, we will first neglect the work-function differences between the metal and SC and other influences such as oxide charge (see Sec. II E).

From Gauss theorem, the electric field in the oxide is constant and given by $E = Q_{SC}/\varepsilon_{ox}$ and, therefore, it is very easy to establish the following relationship:

$$Q_{SC}^s(\psi_s, V_G) = \frac{\varepsilon_{ox}}{t_{ox}} (\psi_s - V_G),$$

with $t_{ox}$ and $\varepsilon_{ox}$ being the oxide thickness and permittivity.

The structure state is thus given by the intersection of this straight line, related to oxide capacitance, with the previous curve (cf. Fig. 2). This kind of graph is a classic one in electronics to solve a transistor-resistor serial system.

We have to remember that the total charge of the device is always equal to zero and check that when the voltage applied to the metal is positive ($V_G > 0$), the metal charge is positive and the SC charge (with $\psi_s > 0$) is negative, like in any classic capacitance.

As shown in Fig. 3, the depletion charge increases versus doping and, therefore, the inversion charge decreases; the SC surface potential $2\psi_b$ slightly increases with doping and is always around 1 V for silicon.

If the SC is N type, we just have to change signs of charges and potentials. This is illustrated in Fig. 4, with various gate voltages. On the same figure, the influence of temperature is demonstrated: we note at low temperature a weak increase of $2\psi_b$ and very steep curves for accumulation and inversion.

**B. MOS capacitance**

Until now, we have established the graphical relationship between the MOS device charges and the applied potential. Therefore, this graph enables us to entirely describe the MOS capacitance characteristic.

In a first approach, using only the $Q_{SC}(\psi_s)$ curve [Fig. 1 or Eq. (1)], we easily understand the behavior of the SC alone, which acts as a capacitance, using $|Q'/\psi_s|$ for the static capacitance and the derivative $dQ/d\psi_s$ for its dynamic value: a maximum is obtained in accumulation and inversion modes, whereas the minimum corresponds to the limit of depletion, just before inversion.

In the flatband zone $\psi_b = 0$, the graph shows a finite SC capacitance. It is a priori a little surprising to have a non-finite SC capacitance since free carriers join the oxide-SC interface: its value is known as being related to the Debye length.$^1$

Obviously, the experimental useful characteristic is the whole MOS capacitance versus the applied voltage $V_G$. In the current literature, this capacitance is given as the oxide capacitance, in series with the SC capacitance. Nevertheless, a direct determination of the whole MOS capacitance can be obtained from the graph, using $|Q_{SC}^s/V_G|$ to obtain the static

$\psi(V)_G$

FIG. 3. (Color online) Doping effect on the SC total charge. In the inversion mode, when doping increases, the surface potential slightly increases, whereas the total SC charge decreases. The main effect is the large increase of the depletion charge. Charges in the oxide would shift the straight line by $-(\varepsilon_{ox}/t_{ox})Q_{SC}^s$ (see Sec. II E).

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FIG. 4. (Color online) Type of SC and temperature effect. A N-type SC is absolutely like a P-type one, with opposite signs for charge and potential (π rotation). In the inversion zone, lowering the temperature increases the absolute value of the surface potential and decreases slightly the total charge.
capacitance and $|\delta Q_{SC}/\delta V_G|$ to get the differential capacitance for the low frequency measurement (Fig. 5).

The MOS structure capacitance versus $V_G$ is clearly close to the oxide capacitance for deep accumulation and inversion and presents a minimum for weak inversion. Obviously, Eq. (1) corresponds to an equilibrium state, and the obtained capacitance behavior is the low frequency one or the capacitance of a MOST, thanks to the electrons coming rapidly from drain and source.

C. Basic $Q\psi$ graph

As the determination of the state of the MOS structure needs to use the complicated $Q_{SC}(\psi_s)$ curve [Eq. (1)], the previous graph is certainly not useful to establish device characteristics. In order to get tractable expressions and to separate fixed and mobile charges, we now have to introduce the simplifications which are commonly used in the equations given by the literature.

In Fig. 2, we can observe that in accumulation as well as in inversion, the surface potential $\psi_s$ changes slowly with the gate voltage; therefore, in order to simplify, the two exponentials are replaced by verticals at $\psi_s=0$ for the holes (accumulation) and at $\psi_s=2\phi_b$ for the electrons (inversion), values for which the argument of their associated exponential become positive. In the inversion situation, the potential $\psi_s$ is then definitely fixed at $2\phi_b$.

For the depletion regime, we keep the square root which corresponds to the acceptor term in Eq. (1), and the charge of the depleted region will then be given by

$$|Q_{dep}^A| = \sqrt{2e_N qN_A} \psi_s.$$  \hspace{1cm} (4)

The main features of the simplified MOS structure are summarized in Fig. 6 (the $Q\psi$V graph). We will denote the maximum depleted zone charge per unit area by $Q_{dep}^A 2\phi_b$ and the inversion charge by $Q_{inv}^A$.

As the graph is very simple, it is easy to obtain the following linear relationship:

$$|Q_{inv}^A| = \frac{e_N}{t_{ox}} (V_G - V_T),$$  \hspace{1cm} (5)

where the threshold voltage $V_T$, the specific value of $V_G$ from which the inversion will be significant, is given by (Fig. 6)

$$V_T = 2\phi_b + \frac{Q_{dep2\phi_b}}{e_{ox}/t_{ox}} = 2\phi_b + \frac{\sqrt{2e_N qN_A} (2\phi_b)}{e_{ox}/t_{ox}}.$$  \hspace{1cm} (6)

Using the maximum depletion depth,

$$I_{2\phi_b} = \sqrt{2e_N} (2\phi_b)/qN_A.$$  \hspace{1cm} (7)

$V_T$ can be expressed using dimensionless ratios,

$$V_T = 2\phi_b \left( 1 + \frac{2e_N t_{ox}}{e_{ox}/I_{2\phi_b}} \right).$$  \hspace{1cm} (8)

This last expression can also be obtained directly from electrostatic considerations, using the serial system of the oxide capacitance ($e_{ox}/t_{ox}$ per unit area) and the static SC depletion capacitance ($2e_{SC}/I_{2\phi_b}$) to divide the gate potential $V_G$: $\psi_s = 2\phi_b$ corresponds to $V_G = V_T$.

D. Nonequilibrium behavior

By extension of the $Q\psi V$ graph, the effect of a rapid voltage increase applied to the gate is also easy to understand. As shown in Fig. 7, at the beginning, there is no inversion layer, and the SC is in deep depletion. Afterward, as time increases, the surface potential $\psi_s(t)$ and the depletion charge $Q_{dep}(t)$ decrease, whereas the total charge $Q_{SC}(t)$ and inversion charge $Q_{inv}(t)$ increase until equilibrium.

From this graph, we can easily determine the required equations to describe the depleted region depth and, therefore, the capacitance evolution; this kind of information is used to characterize the SC material, assuming, for example, that the return to equilibrium is related to the carrier generation in the depleted region or (and) at the interface.
E. Flatband voltage and oxide charge

The relationship $Q_{SC}^A(\psi_s)$ [Eq. (1)] has been established in the so-called ideal case: for $V_G=0$, there is no charge in the metal nor in SC and oxide.

As the metal and the SC have different electronic properties, a work-function difference $\Phi_{ms}$ exists between them, and opposite charges are present in the metal and SC without any applied voltage, like in unbiased $p$-$n$ junctions. Actually, it is necessary to apply a voltage equal to $\Phi_{ms}$ to suppress these charges. On the $Q\psi$ graph, this is equivalent to a shift by $-\Phi_{ms}$ of the straight line $Q_{SC}^A(\psi_s, V_G)$ along $\psi_s$.

In the same way, if charges exist in the oxide, a simple calculation of the electric field in the oxide and its integration along the oxide gives the following relationship:

$$Q_{SC}^A(\psi_s, V_G) = \frac{e_{ox}}{t_{ox}} \psi_s - V_G = \frac{x_C}{t_{ox}} Q_{ox}^A,$$

where $Q_{ox}^A$ represents the total charge per unit area in the oxide, and $x_s$ is the distance from the metal oxide interface to the barycenter of these charges. On the $Q\psi$ graph, we just have to shift the straight line $Q_{SC}^A(\psi_s, V_G)$ along the $Q$ axis by $-x_C/t_{ox} Q_{ox}^A$ (see Fig. 3). This is equivalent to a shift along the $\psi$ axis by $+Q_{ox}^A/(e_{ox}/x_C)$.

As a matter of fact, this shift easily explains how the memories operate: without any applied voltage, for a $P$-type substrate, an inversion layer can be created by means of positive charges located in the oxide, and the most efficient charges are near the SC.

The sum of these two effects is the well-known flatband voltage $V_{FB}$; all ideal equations become correct using $V_G - V_{FB}$ instead of $V_G$ with

$$V_{FB} = \Phi_{ms} \frac{x_C}{e_{ox}} Q_{ox}^A.$$

Obviously, $V_{FB}$ can be affected by other parameters, such as the parasitic interface trapped charge density $Q_{it}^A$.

III. MOST

A. Inversion conditions

We are now able to describe the MOS structure state as a function of nature and thickness of materials and applied voltage $V_G$. However, for a transistor, what will be the effect of the addition of two $N$-type zones on both sides of the MOS structure, the grounded source, and the $V_D$ biased drain? Here, we must refer to a remarkable approach proposed by Refs. 2 and 4, which presented three dimensional graphs of the electrostatic potential $\psi(x, y)$.

- On the source side, no significant change occurs for carriers because there is no applied potential difference between the source and the substrate; therefore, accumulation depletion and inversion occur under the same conditions. The previous $Q\psi V$ graph (Fig. 6) is unchanged.
- On the drain side, there is no change for accumulation and depletion modes because the holes, majority carriers which accumulate or deplete, are managed by the $P$-type material. The inversion mode has a completely different behavior: the drain energy level is decreased by $qV_D$ and, in order to obtain inversion, it is necessary to shift the surface channel potential by $V_D$. Actually, it is necessary to maintain, locally, the same electronic energy level for the channel and the drain. This represents the management of $P$-type SC minority carriers (electrons) by a close $N$-type material, usually described by the quasi-Fermi level concept. In conclusion, on the $Q\psi V$ graph (Fig. 6), we have to move the surface potential of the inversion from $2\psi_0$ to $2\psi_p + V_D$, and finally we get the $Q\psi V^2$ graph (Fig. 8).

This figure shows a weaker inversion charge on the drain side. There are two reasons for this: the electrostatic potential difference between the gate and the substrate is smaller, i.e., $V_G - 2\psi_p - V_D$, instead of $V_G - 2\psi_p$ near the source side, and the depleted zone is larger near the drain side.

B. Drain source current

In inversion mode, we have a conduction current, the electrons being carried away by the local electric field.
\[ I = \mu_n \frac{W}{L} A_{Q \psi} \]

which becomes in the differential form

\[ I \, d\psi = \mu_n W Q_{inv} \, d\psi, \]

The local inversion charge \( Q_{inv} \) is clearly shown in Fig. 9, and the integration from the source to the drain, at distance \( L \), gives

\[ I = \mu_n \frac{W}{L} A_{Q \psi}. \]

The integral in the \( Q\psi \) plane corresponds to the area \( A_{Q \psi} \) delimited by the two verticals \( 2\psi_b \) and \( 2\psi_b + V_D \), the square root of depletion, and the straight line of oxide capacitance (cf. Fig. 9). Finally, the current is given by

\[ I = \mu_n \frac{W}{L} A_{Q \psi} \]

This \( Q\psi V^2 \) graph greatly simplifies the discussion of the various MOST operations and characteristic voltage values.

- For \( V_D \), the pinch-off voltage which cancels the inversion charge near the drain side, the saturation voltage \( V_{D sat} \); if we look at Fig. 10, the relation between the applied potentials is

\[ V_G = 2\psi_b + V_{D sat} + \frac{\sqrt{2e_S q N_N (2\psi_b + V_{D sat})}}{e_{ox}/t_{ox}}. \]

- For \( V_G \), the voltage which cancels the inversion near the source is the threshold voltage \( V_T \). The graph clearly shows that \( V_G \) plays a very similar role to \( V_D \); although not a common approach, to aid understanding, it would be possible to define a \( V_{G sat} \), the maximum gate voltage to have a saturation current for a given \( V_D \) (see Fig. 11). In Eq. (15), we would then replace \( V_G \) by \( V_{G sat} \) and \( V_D \) by \( V_{D sat} \).

After integration to get \( A_{Q \psi} \), we obtained the classical relationships. As an example, for \( V_D < V_{D sat} \), the current is given by

\[ I = \mu_n \frac{W}{L} \left( V_G - 2\psi_b - V_{D sat} \right), \]

in which we have used Eq. (7) to eliminate the doping and clarify the expression.

For \( V_D > V_{D sat} \), the current is no longer dependent on the drain voltage. \( V_{D sat} \) can be calculated using the intersection between the straight line of total charge and the square root of depletion.

\[ V_{D sat} = \frac{2e_S q N_N}{e_{ox}/t_{ox}}. \]
FIG. 12. (Color online) Effect of substrate polarization. Transposition to get a graph similar to the previous $Q\psi/V^2$, i.e., with a grounded substrate: all the voltages are shifted by $-V_{\text{sub}}$ (for a P-type SC, $V_{\text{sub}} < 0$). The source potential is not zero, like the drain in the $Q\psi/V^2$ graph.

C. Subthreshold current

Figure 11 shows also that if $V_G$ is lower than $V_T$, there is only one point of intersection in the depletion mode. The surface potential is thus the same at the source side and the drain side, $\phi_s$ is constant all along the channel, the electric field is null, and consequently the only possible current will be a diffusion current, known as the subthreshold current.

D. Substrate bias effect

In a P-type SC, in order to maintain the source-substrate diode in a reverse state, the bias ($V_{\text{sub}}$) must be negative. An astonishing effect occurs upon application of this negative substrate bias.

- The current decreases, whereas in a simple structure MOS, that is without source-drain, as the effective applied potential across the structure increases, the inversion free carriers of the oxide-SC interface would increase, which should lead a priori to an increase of the current.
- The electric field in the oxide remains unchanged.

This odd behavior can be easily explained within the framework of the $Q\psi$ graph. First, how can we manage to take into account this substrate bias? A solution is to come back to the conditions of the $Q\psi/V^2$ graph (Fig. 8), using a transposition of the problem. To do that, we just remove $V_{\text{sub}}$ from all the applied potentials in order to have a zero biased substrate. Then, we get the graph of Fig. 12.

The only basic difference with the $Q\psi/V^2$ graph is a source voltage which is not grounded. However, this is no longer a problem for us as long as we know how to take into account a drain voltage: the source side is now like the drain side in Fig. 8, with a shift $-V_{\text{sub}}$ positive along the potential axis.

A more useful and comprehensive graph is obtained by a reverse transposition, which is mathematically a simple $V_{\text{sub}}$ shift along the potential axis. We finally obtain a $Q\psi/V^3$ graph (Fig. 13) whose significance is very clear: the verticals $2\psi_s$ and $2\psi_h + V_D$ which correspond to the electrons, controlled by the source and the drain, are at the same $\psi_s$ value as in Fig. 8, but $V_{\text{sub}}$ has a direct effect on the hole population (holes are the majority carriers of the biased substrate) and consequently the accumulation and depletion curves are shifted by $V_{\text{sub}}$.

The surprising current reduction is clearly highlighted due to the increase of the depletion charge, and we can observe in Fig. 13 an increase of $V_T$. Moreover, it appears that in strong inversion, the potential differences in the oxide near the source ($V_G - 2\psi_h$) and the drain ($V_G - 2\psi_h - V_D$) are unchanged (as well as the total charge of the SC); therefore, the oxide electric field does not depend on $V_{\text{sub}}$. This is a remarkable result, especially for physicist, because in the inversion layer, it is possible to change the carrier concentration per unit area, without any change of the local electric field. It is altogether understandable that the surface potential is not affected by the substrate bias because the inversion layer acts as a good conductor, connected to the source and the drain.

Using Fig. 13, in the Ohmic regime, the current under the substrate biased condition is given by the classical relationship

$$I = \mu \frac{W}{L} \frac{V_{\text{ox}}}{t_{\text{ox}}} \left( V_G - 2\psi_h - \frac{V_D}{2} \right) V_D - 4 \frac{\varepsilon_{\text{SC}}}{\varepsilon_{\text{ox}}} l_{2\psi_h} (2\psi_h)^{1/2} (V_D + 2\psi_h - V_{\text{sub}})^{3/2} - (2\psi_h - V_{\text{sub}})^{3/2} \right).$$

This relationship is the most complicated of this presentation. It appears that the $Q\psi$ graph is an elegant way to discuss the effect of the current parameters, think of $V_D$ for example: in the $Q\psi/V^3$ graph (Fig. 13), we just have to shift a vertical line.

E. The most simple graph

In order to validate the simplest graph, we shall first present the most basic implementation for the current rela-
The result for the current relationship is exactly Eq. 16. We shall use the two well known formulas:

- \( Q = CV \), where \( Q \) is the charge, \( C \) the capacitance, and \( V \) the potential applied to the capacitance;
- \( U = RI \), Ohm’s law, where \( U \) is the potential that induces the free carriers drift, \( R \) the resistor, and \( I \) the current.

Within the framework of our study, \( R \) can be expressed as

\[
R = \frac{L}{S} = \frac{1}{q n \mu_n t_{\text{inv}}} W.
\]

Therefore,

\[
I = \mu_n q n t_{\text{inv}} \frac{W}{L} V_D. \tag{19}
\]

In \( q n t_{\text{inv}} \), we can recognize the charge per unit area of the inversion layer. Its simplest expression could be

\[
\frac{Q}{LW} = \frac{e_{\text{ox}}}{t_{\text{ox}}} V_G. \tag{20}
\]

However, in order to take into account the depletion charge, we must use Eq. (5). At this stage, the current is given by

\[
I = \mu_n \frac{W e_{\text{ox}}}{L t_{\text{ox}}} (V_G - V_T) V_D. \tag{21}
\]

To complete this approach, we have to remember that the capacitance cannot be treated as usually because by the SC source, the applied potential is zero, whereas it is \( V_D \) close to the drain side. As a first approximation, we can consider that the mean value of the voltage applied across the capacitance is therefore \( V_G - V_D/2 \) instead of \( V_G \). Finally, we obtain

\[
I = \mu_n \frac{W e_{\text{ox}}}{L t_{\text{ox}}} (V_G - V_T - V_D/2) V_D. \tag{22}
\]

Now, let us consider the \( Q \phi \) method. We would expect a simplified graph. As mentioned above when we use \((e_{\text{ox}}/t_{\text{ox}})(V_G - V_T - V_D/2) \) for the inversion charge, we assume a constant threshold \( V_T \) for the inversion, which means a constant depletion charge all along the channel, equal to the charge near the source, whose depth is \( l_{2\theta_b} \) [see Eq. (7)]. As a matter of fact, on the \( Q \phi \) graph, we just have to replace the square root of the depletion charge by a constant from the source to the drain, see Fig. 14.

Finally, to derive the current, we need to calculate the area \( A_{Q\phi} \) which has the shape of a trapezium. More physically, this area is a rectangle minus a triangle: the rectangle represents the current for a constant inversion charge per unit area, and the triangle the decreasing of the inversion charge due to \( V_D \), which introduces the quadratic term \(- (V_D/2) V_D\). The result for the current relationship is exactly Eq. (22).

Of course, from this simplified graph, it is very simple to explain the current behavior as a function of \( V_D \) and \( V_G \).

Initially, the current increases as \( V_D \) rises and is named the Ohmic current; its value is given by Eq. (22). The saturation voltage is given by \( V_{D\text{sat}} = V_G - V_T \) (see Fig. 14) and the corresponding current is called saturation current. This saturation voltage \( V_{D\text{sat}} \) is slightly lower than \( V_D \) because the depletion charge is now smaller. For this drain potential, the charge near the drain is null (and \( \partial I / \partial V_D = 0 \)). The saturation current corresponds to the area of a triangle, and we get

\[
I_{\text{sat}} = \frac{1}{2} \mu_n \frac{W e_{\text{ox}}}{L t_{\text{ox}}} (V_G - V_T)^2. \tag{23}
\]

Now, if we change the gate voltage \( V_G \),

- as usual, there is no current for \( V_G < V_T \), except a subthreshold current, as said previously;
- when \( V_G \) increases from \( V_G = V_T \) to \( V_G = V_T + V_D \), the current increases quadratically with \( V_G \) (the transistor is in the saturation mode);
- for \( V_G > V_T + V_D \), the current increases linearly with \( V_G \).

From Fig. 14, the current behavior as a function of the substrate bias \( V_{\text{sub}} \) can be established without any difficulty, using the graph modification previously explained in Sec. III D: the accumulation and depletion curves, which are related to the majority carriers of the biased substrate, have to be shifted by \( V_{\text{sub}} \) along the \( \phi \) axis.

It is well known that relationship (22) can be obtained from Eq. (16) using a limited development, assuming that the ratio \( V_D/2 \phi_b \) is small. Obviously, knowing that \( 2 \phi_b = 1 \) V, this ratio can be rather high, but the simplified current [Eq. (22)] is often close enough from the exact current [Eq. (16)] to be very useful. The graph of Fig. 14 shows that this limited development is equivalent to a constant depletion charge between the source and drain. Besides, it allows an easy estimation of the relative error resulting from the limited development: it is just, from the source to the drain, the area between the square root of depletion charge and the constant depletion charge compared to the current area \( A_{Q\phi} \).

IV. CONCLUSION

The benefits of the approach using the \( Q \phi \phi^2 \) graphs are twofold. First of all, this kind of graph enables an easy visu-

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**FIG. 14.** (Color online) Approximation for the \( Q\phi \phi^2 \) graph. A constant depletion charge is assumed from the source to the drain. The graph emphasizes the role of the drain voltage and allows a very simple calculation of the most useful relationship for the current. To determine \( V_{D\text{sat}} \) we just have to notice that when the inversion charge near the drain is null, \( A_H = V_G - V_T = V_D \).
alization of the impact of the material and electrical parameters on the MOS structure and MOST behavior: think of $V_D$, for example, in Eq. (17). Secondly, the physical reasons for these behaviors are highlighted, and the relevance of the frequently used approximations, which lead finally to simple relationships, can be easily evaluated.

Obviously, the use of such a graphical representation does not imply a qualitative approach: it is noted that the classical equations of the literature can always be directly deduced from the graphs. Therefore, for the visualization of the main MOS structure and MOST equations as a function of the impacting parameters, the use of such graphical techniques is both a valid and an attractive approach.

3Additional unpublished information can be found at http://pagesperso-orange.fr/physique.belledonne/